Journal of Systemics, Cybernetics and Informatics (2025) 23(6), 1-5
https://doi.org/10.54808/JSCI.23.06.1

A Markov Chain Approach for Modelling Normally Distributed Online Assessment Time in a University Setting

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ABSTRACT

The Markov chain (MC) technique is applied to an online invigilated assessment situation to predict the writing time in a typical university setting. Different students' writing times cannot be determined accurately in advance and are associated with randomness. This preliminary study simulates data related to the time to download the question paper and the writing time, from a normal distribution. The time variable is simulated to have a reasonably good approximation of the real settings where most students' writing times are spread around the expected value, namely the mean. Simulations are conducted based on the experience and knowledge of researchers in the online teaching and learning environment. Computer simulations demonstrate that the writing time estimates depicted a stable convergence, thus giving clear insights for optimising online assessment implementation. The findings showed that the average writing time of a selected trial reaches a stable value at 1.498 hours (89 minutes) within the confidence interval [0.6, 2.5], at 95%. Therefore, these results offered a more realistic range of feasible times to guide academic practitioners on the planning and implementation of invigilated online assessments.

Keywords: Markov chain, stochastic process, modelling, assessment writing time, invigilated assessment, stable convergence.

1. INTRODUCTION

Students sometimes face challenges in complying with the exact time required to complete a given assessment easily. These challenges range between the speed of writing an assessment and its completion duration, which is constrained by the university. This situation significantly worsens when online exams or tests are administered through computer systems. Issues could be the slowness of the network or even its breakdown, load shedding, or the invigilation app not responding adequately. The writing times for a specific assessment become uncertain, hence random. From the researchers' instructional practice in online test/exam settings, the time for an online test is 2 hours, with 15 minutes before the exam to download the question paper and 30 minutes to upload the answers or the actual script. Information Record In System (IRIS) software packages have been used for online tests/exams. IRIS is a machine-based learning software that records audio, video, and monitors activities to detect possible academic cheating during an assessment. It is stressed that assessments constitute a vital means to judge students' competency but are insufficient to reflect student graduateness. They make a powerful teaching and learning component to estimate students' pass rates. Hence, assessments are regarded as a critical guiding element for a university to align administrative and academic aspects to enhance learning.

Assuming a deterministic writing time for an invigilated online synchronous assessment appears inaccurate. This implies that the writing time is associated with probabilistic, naturally stochastic characteristics. Stochastic processes can be explored to model real-time writing times of given assessments. Using elegantly suitable methods to model the writing time is very appealing. For that, the importance of the Markov Chain cannot be overstated, as it plays a crucial role in a wide range of applications across various fields. e.g. estimating delay times between trains [1], traffic flow control [2], estimating surgery time [3], time value in information of animal populations [4], safety maritime assessment [5], student performance through examination [6], analysis of biological data [7].

Despite its popularity, the MC has been rarely applied in the time estimation of an online assessment, specifically occurring synchronously. Therefore, the focus of this study is threefold, i.e. to propose a Markov chain model for an invigilated online assessment setting, to simulate essentially writing time using normally distributed durations/times, and to evaluate the level of convergence of the assessment mean writing time over a repetition of trials. From this, the need arises for modelling tools that describe typical assessment writing time within the acceptable margins of variance. This could make students at ease during actual assessment implementation, as far as time is concerned. The remainder of the paper is structured as follows: First, a brief introduction to the Markov Chain is provided. Next, the methodological approach that supports this study is outlined. Following that, the results of the MC simulations are discussed. Finally, the conclusion summarizes the key findings.

2. OVERVIEW OF MARKOV CHAINS

The simplicity of Markov chains (MC) is founded on the premise that the future state of a system depends only on the current state, without considering the previous states. The time in MC processes is supposed to be continuous, occurring randomly [8]. It is important to note that there is always a transition from one state to the next in the MC. This transition is generally described by transition probabilities, summarised in a transition matrix. It was highlighted earlier that the literature has shown diverse applications of the MC to real-life problems.

From the foregoing, the MC process comprises a set of states and probabilities, which move from state “a” to state “b”. A transition from one state to itself may occur since a system may remain in the same position. Therefore, the following equation gives the probability of a random variable X moving from state b to state a:

Q_{ab} = P(Xt+1 = b |Xt = a) for a, b ∈ B, t = 0,1,2,..., (1)

where the transition matrix of MC is defined by the following

$$A = [Q_{ab}] \quad (2)$$

The transition matrix is essential since it helps calculate the probability distribution at different times.

The journey of a system at different states defines the trajectory of a Markov chain [9]. In the trajectory, it is possible to have a state where the system gets stuck once it enters, or an absorbing state of an MC [10] and such a state has a probability of 1. The probability of absorption defines the likelihood that the system will attain the absorbing state [10]. Conversely, a non-absorbing or transient state differs from the absorbing state. Since the absorbing state is a permanent situation, a non-absorbing state may move to an absorbing state, but not vice versa. When the transition probability becomes stationary, steady, and characterised by an invariant time, the MC is said to be ergodic. Ergodicity helps to assess the long-term behaviour [11] related to changing writing times of online writing times for online assessments. The stationary distribution of a finite Markov chain is the summation of certain normal distributions [12].

3. METHODOLOGY

Defining Markov Chain states

Three central states were defined for the online assessment implementation: downloading the paper, writing the time, and uploading it to the portal after the assessment has finished during the regulated time.

Methodological approach

The rationale of using MC is that the probability of being in the state of writing the paper only depends on the fact that the student is in the state of downloading the paper. Thus, it is trivial for the student to write a test or exam, unless the paper has been downloaded. In the same way, the student will upload the script, unless they have finished writing the paper. The question here is not whether the student answered all the questions. The verb “Finish” refers to the time allocation or duration of the assessment. From the researchers’ experiences in their academic environment, the initial state, the downloading state, is set to 15 minutes maximum, and the second state is the writing time, which is the longest of them all and is set to a maximum of 2 hours. Typically, the uploading stage is approximately 30 minutes and is considered part of the test completion. Hence, it is assumed there is no more writing time during the upload process. The writing time can be taken to follow a normal distribution since the completion time can be spread around the mean. The standard deviation value is usually a fraction of the mean. Thus, the standard deviation values were associated with states, namely D and W, for which the standard deviations were set arbitrarily to carry out simulations. The above can be summarised in Table 1 below.

Table 1. Characteristics of writing time considerations for running Markov Chain simulations (μ, σ: mean and standard deviation)

Defining State	**Timing distribution**	**Statistical Parameters**	**Writing Times**
D: Paper download	Normal	μ = 0.167 h σ = 0.02 h	1-15 min
W: Writing time	Normal	μ= 1.5 h σ= 0.5 h	1-2 hours
L: Finishing State	Uniform (t =0)	μ = 0 σ= 0	0

Due to the characteristics of the last state, as shown in Table 1, it did not play a role in estimating writing time during the simulation time. Python code in Google Colab was used to implement the MC model. Figure 1 shows the MC structure of the situation at hand and illustrates that the second state is the transition between the first and the finishing state after the assessment time has expired.
The simulation of the MC algorithm can be summarised in the following steps:

- Download time is initialised
- Set the sample state timing D spent in each state
- Writing time is accumulated when in the W state
- Repeat the above until the L state is reached
- Several simulations (iterations) are run for writing time estimate stability

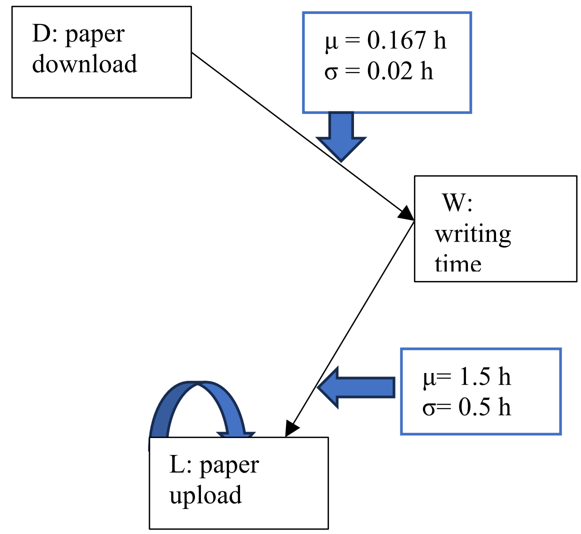

Fig. 1. Markov Chain structure of writing timing of an online invigilated test/exam.

This final stage is considered absorbing as the student is trapped once the finishing time is reached. Outlier cases are not considered, where the student starts writing once the paper is downloaded, for example, after 5 or 10 minutes of the initial state. Or it is also possible that the student had a challenge downloading the paper within the prescribed time and started the test later. Another situation is that a student may still use a few minutes to complete a given question during the uploading time.

4. RESULTS AND DISCUSSION

Convergence for writing time

The time parameters for the different states were set as shown below.

```
# States and time parameters

# States
states = {
    'D': 'Downloading exam paper',
    'W': 'Writing IRIS invigilated exam',
    'F': 'Exiting the IRIS invigilated exam'
}

# Time parameters (in hours)
time_params = {
    'D': {'mean': 0.167, 'next_states': ['W']},  # Avg 0.167 hr downloading paper
    'W': {'mean': 1.5, 'next_states': ['F']},  # Avg 1.5 h writing exam
    'F': {'mean': 0.0, 'next_states': []}      # Absorbing state = finishing writing
                                                          # time
}
```

As displayed in Figure 2, the output result for the writing time showed numerical stability as the number of simulations increased. Hence, the solution converged around 1.5 hours. In this way, the student reached the absorbing state, i.e. finishing/completion time. Such an MC was confirmed to be characterised by ergodicity since the stationary state could be reached.

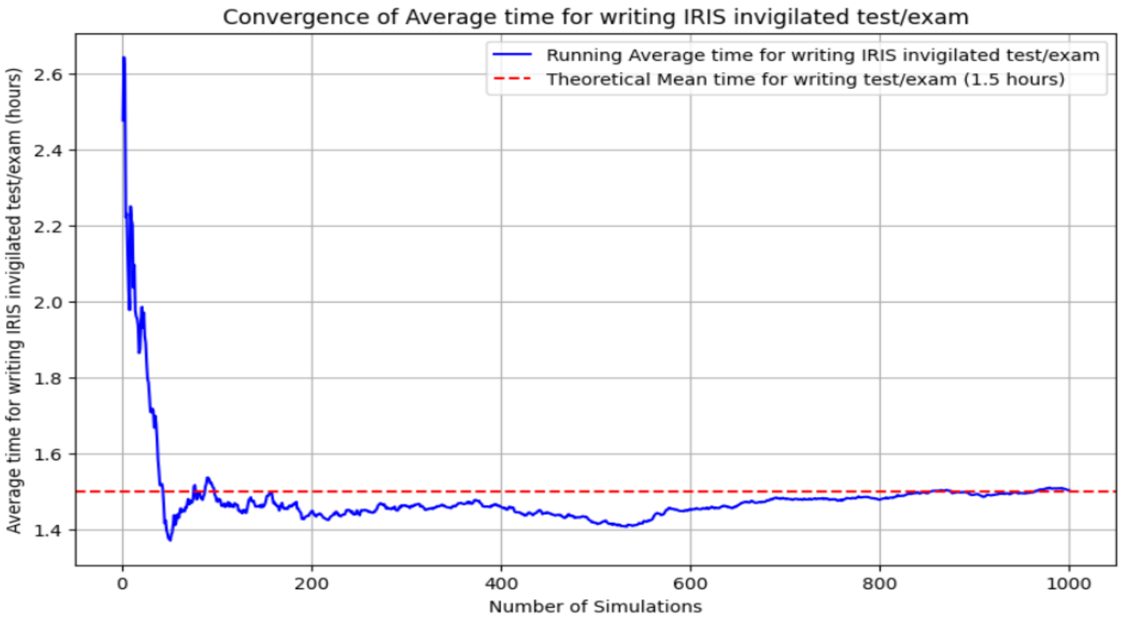

Figure 2. Simulations for the average time of an online IRIS invigilated assessment.

Statistical outputs

The same states and time parameters, as defined previously, were used. The statistical results are summarised in Figure 3. The histogram displays the actual frequency distribution of simulated writing time. The probability density has been fitted to the histogram to create a continuous random variable for the writing time and constitutes the theoretical normal distribution. This probability density had $\mu = 1.5$ hours, $\sigma = 0.5$ hours and was simply truncated at the initial time. The writing time mean and median were close to 1.5 hours, which could validate the assumption of a normal distribution. For this value, the theoretical probability is 80%, as shown in Figure 3. Each writing time is associated with a frequency or a likelihood of occurrence. Thus, the probability that a student completes the test /exam in 1 h and 2 h is approximately 40% and 50%, respectively.

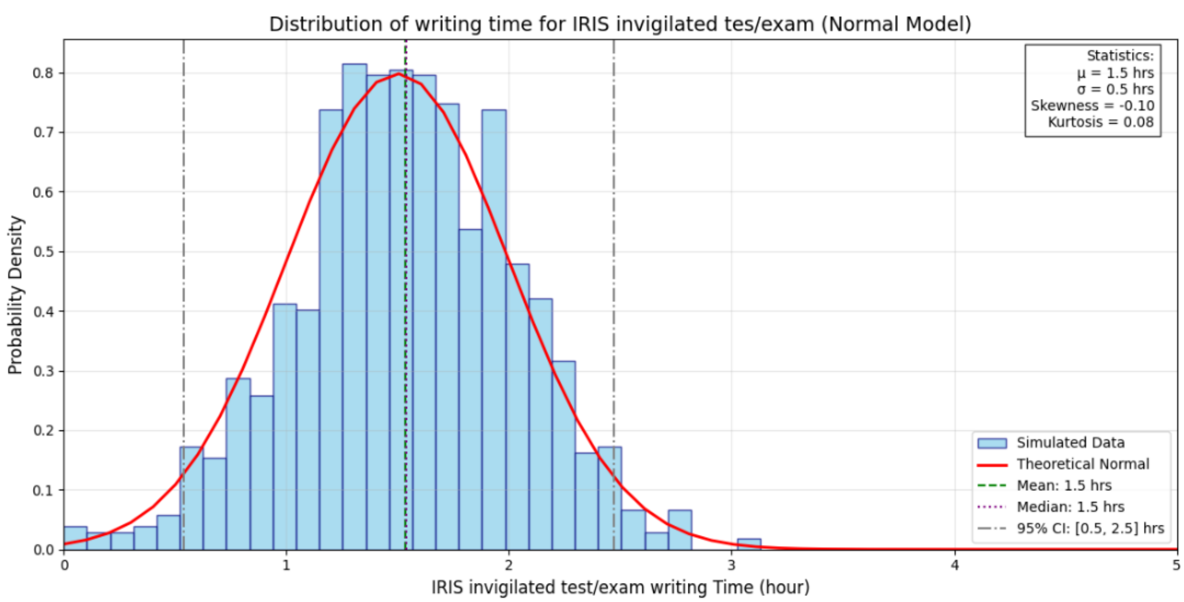

Figure 3. Probability distribution for writing timings of an online assessment.

Below is the writing time during a trial

Average Writing IRIS invigilated test/exam time: 1.498 hours (~89.9 minutes)

The skewness coefficient was not significantly different from 0, and the kurtosis displayed peakedness compared to the normal distribution. The 95% confidence interval limits were 0.6 and 2.5 hours. This visualisation of results is beneficial for academic managers to provide both the writing time expectations and the interval of possible timing outcomes. This could facilitate decisions on assessment implementation time by serving better students.

5. CONCLUSIONS

This preliminary study has simulated the application of the Markov chain to estimate the writing time of online assessments in an invigilated environment. This was limited to a situation of normally distributed time. The study attempted to derive a realistic/acceptable time interval that matched the students' writing time expectations. In this way, university managers, academics and administrative staff should look seriously at the setting of the assessment timings.

The probability trajectory and distribution density of writing times helped determine possible finishing times. Hence, the average time for writing an invigilated assessment was between 0.6 and 2.5 hours. Subsequently, one could reach a stage where the probability approaches zero. Specific invigilated assessments may require different writing times (or time allocation). This could be justified by the writing time interval derived from the MC simulations.

A survey should be established to have insight into the students' opinions on the sufficiency of time allocation for different online assessments. The test timing is the same for all modules in the current practice. The current study has the merit of showing the stochastic predictive capability of MC for the online writing test/exam. The range of times showed that this variable should not be considered deterministic but should be associated with a certain degree of randomness for students to complete their assessments. Students do not always spend the same time writing an invigilated assessment. Nonetheless, lower and upper limits have been estimated for the writing time.

From this study, universities can reflect on the implementation of assessments in terms of completion time by the students. This may affect, to a lesser degree, the credit estimation of different courses. The Markov chain method provides a quantitative framework for re-evaluating assessment time allocation. In this way, decision makers in the academic environment could make an informed determination on this allocation. As simplified in this study, the methodological approach could be tested on actual data obtained from students' opinions. Further investigation could include other statistical distributions.

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