Off-Axis Gaussian Beams with Random Displacement in Atmospheric Turbulence

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ABSTRACT

Our recent work in which we study the propagation of the general Hermite-sinusoidal-Gaussian laser beams in wireless broadband access telecommunication systems is elaborated in this paper to cover the special case of an off-axis Gaussian beam. We mainly investigate the propagation characteristics in atmospheric turbulence of an off-axis Gaussian beam possessing Gaussian distributed random displacement parameters. Our interest is to search for different types of laser beams that will improve the performance of a wireless broadband access system when atmospheric turbulence is considered. Our formulation is based on the basic solution of the second order mutual coherence function evaluated at the receiver plane. For fixed turbulence strength, the coherence length calculated at the receiver plane is found to decrease as the variance of the random displacement is increased. It is shown that as the turbulence becomes stronger, coherence lengths due to off-axis Gaussian beams tend to approach the same value, irrespective of the variance of the random displacement. As expected, the beam spreading is found to be pronounced for larger variance of displacement parameter. Average intensity profiles when atmospheric turbulence is present are plotted for different values of the variance of the random displacement parameter of the off-axis Gaussian beam.

Keywords: Free Space Optics, Atmospheric Turbulence, Laser Beam Propagation, Off-Axis Beams.

1. INTRODUCTION

It is known that the off-axis-Gaussian beams are special form of Gaussian beams in which complex displacement parameters are introduced. It is thus possible to obtain source coordinate dependent attenuation and phase at the exit plane of the laser. In this work we have incorporated the complex displacement parameters as Gaussian random variables. In this manner we mimic the spatial partial coherence property and apply it to an off-axis-Gaussian laser beam wave source. Propagation of such excitation is then examined in a turbulent atmosphere.

Sometimes named also as the decentered Gaussian beams, the off-axis-Gaussian beams are studied by many researchers both at the excitation plane and after having propagated in various optical systems [1]-[5]. We have recently introduced off-axis-Gaussian beams in a turbulent atmosphere [6], [7]. Our analysis in these studies is based on deterministic complex displacement parameters at the source plane representing the attenuation and phase shift. Our motivation is to search whether some features of the off-axis-Gaussian beams such as the displacement, coherence length and the intensity profiles at the receiver plane can be utilized in certain applications in atmospheric optical communication links. Also we want to understand how these features are influenced, when the complex displacement parameter of the off-axis-Gaussian beam varies in a random manner.

2. FORMULATION

In the presence of atmospheric turbulence, the field at the receiver plane due to an off-axis-Gaussian beam excitation is given by the extended Huygens Fresnel formula as

$$u\left(\mathbf{p}, z = L\right) = \frac{k \exp(ikL)}{2\pi i L} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{d}^2\mathbf{s}\, u\left(\mathbf{s}, z = 0\right) \times \exp\left[ik\left(\mathbf{p} - \mathbf{s}\right)^2 / \left(2L\right) + \psi\left(\mathbf{s},\mathbf{p}\right)\right], \quad (1)$$

where $\mathbf{s} = \left(s_x, s_y\right)$ and $\mathbf{p} = \left(p_x, p_y\right)$ are the transverse source and transverse receiver coordinates, respectively, $i = (-1)^{1/2}$, and z is the propagation axis, i.e., $z = 0$ and $z = L$ present the source and the receiver planes. L is the link length, k is the wave number, $\psi\left(\mathbf{s},\mathbf{p}\right)$ is the solution to Rytov method representing the random part of the complex phase of a spherical wave propagating from the source point $\left(\mathbf{s}, z = 0\right)$ to the receiver point $\left(\mathbf{p}, z = L\right)$. In writing Eq. (1), time dependence of the field is not included. $u\left(\mathbf{s}, z = 0\right)$ denotes the source field distribution of an off-axis-Gaussian beam given by

$$u(\mathbf{s}, z=0) = A \exp\left[-\frac{1}{2}\left(\frac{s_x^2}{\alpha_{sx}^2} + \frac{s_y^2}{\alpha_{sy}^2}\right)\right]$$
$$\times \exp\left[-i\left(V_x s_x + V_y s_y\right)\right] \exp(-i\phi), \qquad (2)$$

where A is the amplitude of the field at the origin of the source plane (i.e. at $s_x = s_y = z = 0$) taken as unity in our calculations, ϕ is the constant phase factor, α_{sx} and α_{sy} are the source size of the Gaussian beam in s_x and s_y directions, V_x and V_y are the complex displacement parameters associated with the Gaussian part of the beam in the s_x, s_y directions, respectively. In general, $V_x = V_{xr} + iV_{xi}$, $V_y = V_{yr} + iV_{yi}$, where V_{xr}, V_{xi} are the real and imaginary components of V_x, and V_{yr}, V_{yi} stand for the real and imaginary components of V_y. In this paper, the complex displacement parameters are taken to be real, equal in both directions and also are assumed to be Gaussian distributed random variable with zero mean and the variance denoted by σ_V^2. In our results, we have also taken equal source sizes in both directions, i.e., $\alpha_{sx} = \alpha_{sy} = \alpha_s$. In Eq. (2), focal lengths in both transverse source coordinates are taken as infinity, i.e., collimated excitation is considered.

Second order mutual coherence function evaluated at the receiver plane in the turbulent medium is given by

$$\Gamma_2(\mathbf{p}_1, \mathbf{p}_2, z=L) = \langle u(\mathbf{p}_1, z=L) u^*(\mathbf{p}_2, z=L) \rangle, \qquad (3)$$

where $\mathbf{p}_1$ and $\mathbf{p}_2$ are two different points at the receiver plane, $< \;>$ denotes the ensemble average over the statistics of the complex displacement parameter and turbulence where these two statistics are assumed to be independent. Substituting Eq. (2) into Eq. (1), and inserting the resulting receiver field in Eq. (3), applying the receiver coordinate transformation such that $\mathbf{p}_c = 0.5(\mathbf{p}_1 + \mathbf{p}_2)$, $\mathbf{p}_d = \mathbf{p}_1 - \mathbf{p}_2$ and performing the integrations over the source coordinates by using Eq. 3.323.2 of Ref. 8, Eq. (3) is evaluated for two cases, when $\mathbf{p}_c = 0$ and $\mathbf{p}_d = 0$. We present below these two cases separately. The first case when $\mathbf{p}_c = 0$ is found as

$$\Gamma_2(\mathbf{p}_d, z=L) = \frac{1}{\left[1 + \left(\frac{L}{k\alpha_s^2}\right)^2 + \left(\frac{L\sigma_V}{k\alpha_s}\right)^2 + \left(\frac{2L}{k\alpha_s\rho_0}\right)^2\right]}$$
$$\times \exp\left[-\left(\frac{\mathbf{p}_d}{r_0}\right)^2\right], \qquad (4)$$

where r_0 is the coherence length at the receiver plane which is calculated to be

$$r_0 = \rho_0 \left[\frac{1 + \left(\frac{L}{k\alpha_s}\right)^2 \left(\frac{1}{\alpha_s^2} + \frac{4}{\rho_0^2} + \sigma_V^2\right)}{3 + \left(\frac{L}{k\alpha_s}\right)^2 \left(\frac{1}{\alpha_s^2} + \frac{3}{\rho_0^2} + \left(\frac{\sigma_V \rho_0}{2}\right)^2 + \sigma_V^2\right)}\right]^{1/2}, \qquad (5)$$

Here $\rho_0 = \left(0.545\, C_n^2 k^2 L\right)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent medium, C_n^2 being refractive index structure constant.

The second case when $\mathbf{p}_d = 0$ is found as

$$\Gamma_2(\mathbf{p}_c, z=L) = \frac{1}{\left[1 + \left(\frac{L}{k\alpha_s^2}\right)^2 + \left(\frac{L\sigma_V}{k\alpha_s}\right)^2 + \left(\frac{2L}{k\alpha_s\rho_0}\right)^2\right]}$$
$$\times \exp\left[-\left(\frac{\mathbf{p}_c}{\alpha_b}\right)^2\right], \qquad (6)$$

where α_b is the beam width at the receiver plane which is found as

$$\alpha_b = \alpha_s \left\{1 + \left(\frac{L}{k\alpha_s^2}\right)^2 \left[1 + \left(\frac{2\alpha_s}{\rho_0}\right)^2 + \alpha_s^2 \sigma_V^2\right]\right\}^{1/2}, \qquad (7)$$

Expression in Eq. (6) also yields the average intensity profile at the receiver plane due to an off-axis-Gaussian beam excitation with random displacement parameter in turbulence. Thus, re-writing Eq. (6) for the average receiver intensity, we have

$$\langle I_r(\mathbf{p}) \rangle = \frac{\exp\left[-\left(\frac{\mathbf{p}}{\alpha_b}\right)^2\right]}{\left[1 + \left(\frac{L}{k\alpha_s^2}\right)^2 + \left(\frac{L\sigma_V}{k\alpha_s}\right)^2 + \left(\frac{2L}{k\alpha_s\rho_0}\right)^2\right]}, \qquad (8)$$

where $\langle I_r(\mathbf{p}) \rangle$ denotes the average intensity at an arbitrary receiver coordinate, i.e., at $\mathbf{p}_c = \mathbf{p}$.

Our formulas provided by Eqs. (4) and (6) correctly reduce to the average intensity expression in turbulence for Gaussian beam wave given in Ref. 9 in the limit when $\sigma_V^2 = 0$.

3. RESULTS

All our results are obtained at the wavelength of $\lambda = 1.55\mu$m, since this is the most widely used wavelength in the currently operational free space optical access communication links. Coherence length r_0 shown in the figures has the unit of meters. In Fig. 1, coherence length at the receiver plane given by Eq. (5) is plotted versus the refractive index structure constant for various values of the variance of the displacement parameter. It is seen that for a fixed turbulence strength, the coherence length calculated at the receiver plane decreases as the variance of the random displacement is increased. As the turbulence becomes stronger, coherence lengths due to off-axis Gaussian beams tend to approach the same value, irrespective of the variance of the random displacement.

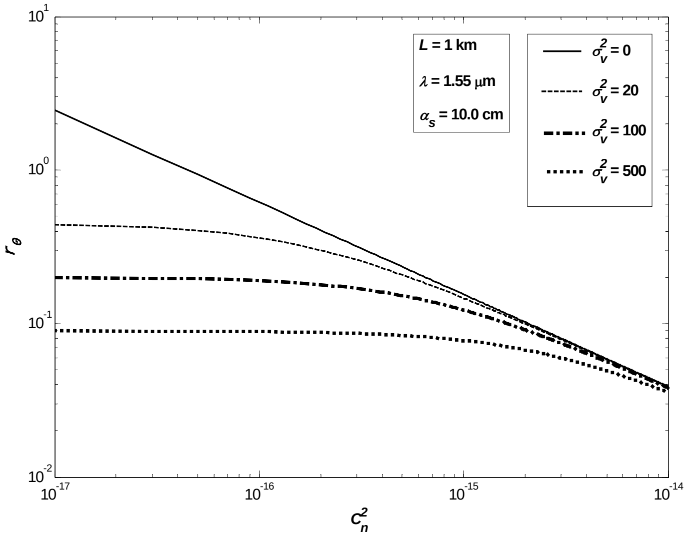

Fig. 1. Coherence length at the receiver plane versus the structure constant for various values of the variance of the displacement parameter.

The change of the coherence length versus the refractive index structure constant for various values of the variance of the displacement parameter is shown in Fig. 2 for a smaller source size than that of Fig. 1. Evaluation at a fixed propagation distance reveals that the spatial coherence length decreases as the strength of atmospheric turbulence increases, and this is valid for all values of the variance of the random displacement parameters of a fixed sized source. However, for the same turbulence levels, coherence length is smaller for larger variance of the random displacement parameter, eventually leading to similar coherence lengths for sufficiently strong turbulence levels. Comparing Fig. 1 with Fig. 2, we observe that the smaller size source yields larger coherence length at the same turbulence strength. Also, when the source size is smaller, the variance of displacement parameter becomes less effective.

The change of the coherence length versus the link length for various values of the variance of the displacement parameter is shown in Fig. 3. Evaluation at a fixed source size reveals that the spatial coherence length decreases as the link length increases, and this is valid for all values of the variance of the random displacement parameters of a fixed sized source. Again, for the same turbulence levels, coherence length is smaller for larger variance of the random displacement parameter, eventually leading to similar coherence lengths for sufficiently long link lengths.

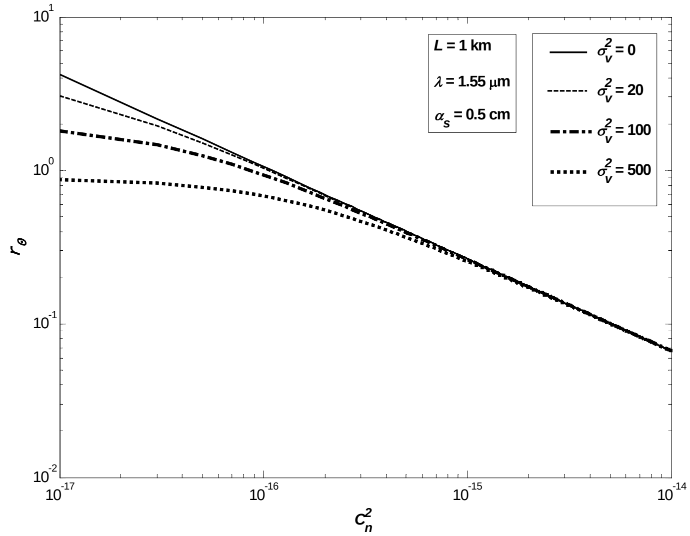

Fig. 2. Coherence length versus the structure constant for various values of the variance of the displacement parameter for a smaller source size.

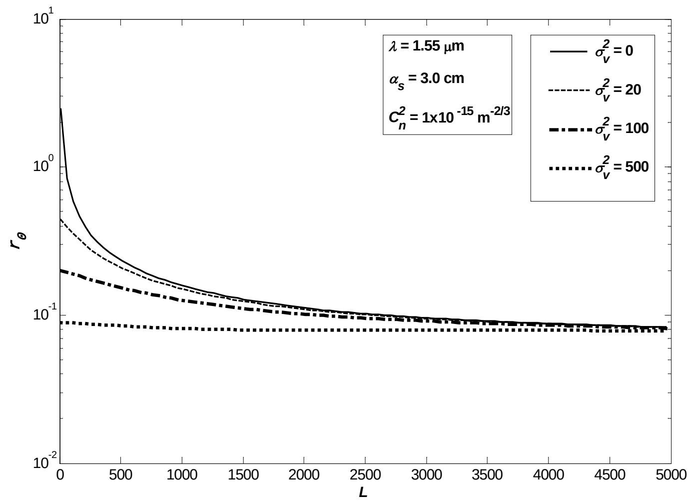

Fig. 3. Coherence length versus the link length for various values of the variance of the displacement parameter.

In Fig. 4, the coherence length is plotted versus the structure constant at a fixed variance of displacement parameter and for various source sizes. Examining Fig. (4), we find that in a similar manner to Fig. (2), the spatial coherence length decreases as the strength of atmospheric turbulence increases which is valid for all values of source sizes having a fixed variance of the random displacement. This time, for the same turbulence levels, coherence length is smaller for larger source sizes, eventually leading to similar coherence lengths when turbulence attains relatively large structure constants.

In Fig. 5, beam width at the receiver plane given by Eq. (7) is shown versus the link length for various values of the variance of the complex displacement parameter and in the absence of atmospheric turbulence ($C_n^2 = 0$).

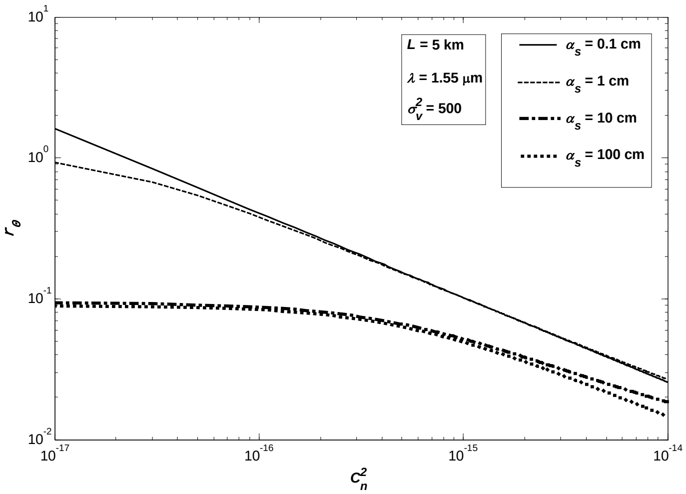

Fig. 4. Coherence length versus the structure constant for various values of the source size with a fixed variance of displacement parameter.

Fig. 6 uses the same parameters as in Fig 5 except that turbulence is included. As expected, the beam spreading becomes pronounced for larger variance of displacement parameter. This is attributed to the fact that the off-axis Gaussian beam wave having a Gaussian distributed random displacement parameter has similar characteristics as that of a spatially partially coherent laser beam. By comparing Figs. 5 and 6, we also note that the beam spreads more in turbulence than in free space when the other source and medium parameters are kept the same.

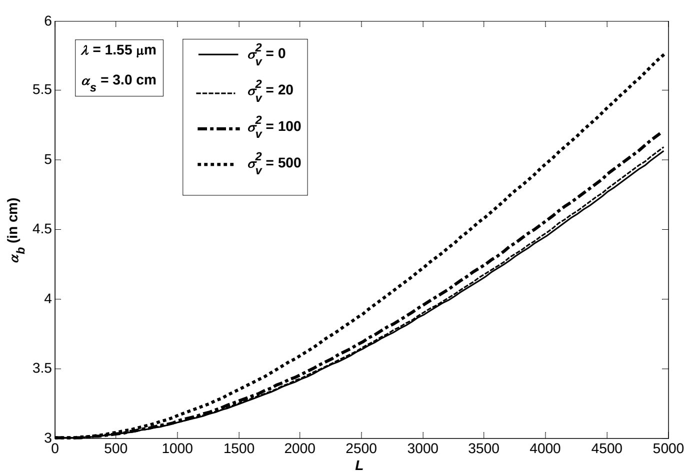

Fig. 5. Beam width versus the link length in the absence of turbulence for various values of the variance of the displacement parameter.

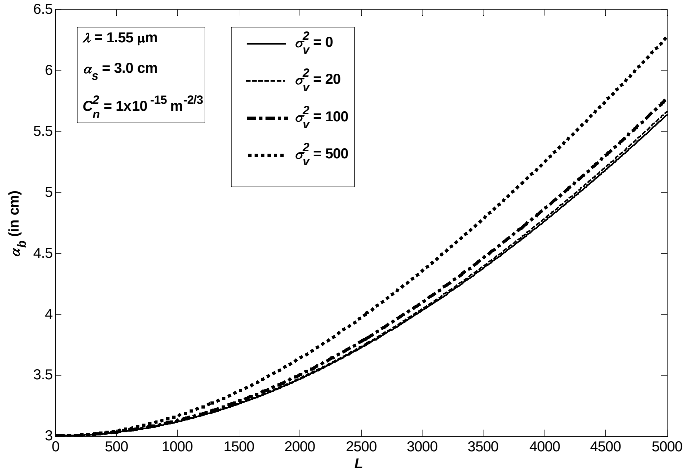

Fig. 6. Beam width versus the link length in turbulence for various values of the variance of the displacement parameter.

The normalized average receiver intensity profiles in atmospheric turbulence are plotted in Fig. 7 for different values of the variance of the random displacement parameter of the off-axis Gaussian beam. The normalized average receiver intensity is found by dividing the average receiver intensity given in Eq. (8) by the peak value of average receiver the intensity at $\sigma_v^2 = 0$. The normalized average receiver intensity profile becomes smaller along the slanted receiver axis as the variance of the displacement parameter increases. This reduction is particularly pronounced at the origin of the receiver plane.

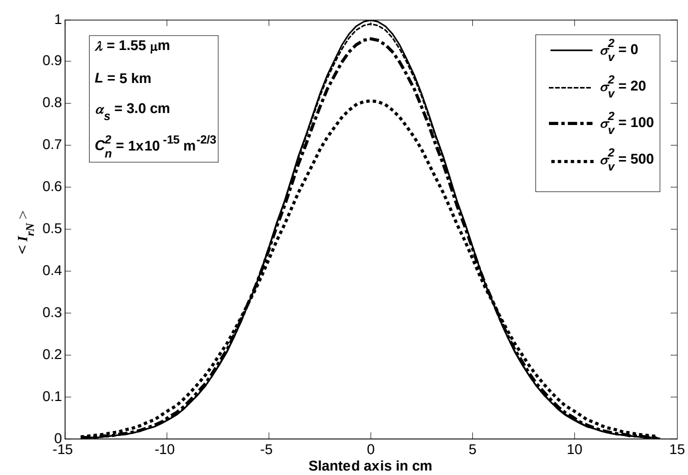

Fig. 7. Normalized average receiver intensity profiles in turbulence for different variance of displacement parameters

4. CONCLUSIONS

We examined the propagation characteristics in atmospheric turbulence of an off-axis Gaussian beam possessing Gaussian distributed random displacement parameters. For a fixed turbulence strength, the coherence length calculated at the receiver plane decreases as the variance of the random displacement is increased. As the turbulence becomes stronger, coherence lengths due to off-axis Gaussian beams tend to approach the same value, irrespective of the variance of the random displacement. For a smaller source size, the same behaviour is observed except that the smaller size source yields larger coherence length at the same turbulence strength. Also, the variance of displacement parameter becomes less effective for smaller source sizes. For a fixed source size, the spatial coherence length decreases as the link length increases, and this is valid for all values of the variance of the random displacement parameter. For the same turbulence levels, coherence length is smaller for larger variance of the random displacement parameter, eventually leading to similar coherence lengths for sufficiently long link lengths. Also, at a fixed propagation distance, the spatial coherence length decreases as the strength of atmospheric turbulence increases which is valid for all values of source sizes having a fixed variance of the random displacement. This time, for the same turbulence levels, coherence length is smaller for larger source sizes, eventually leading to similar coherence lengths when turbulence attains relatively large structure constants.

As expected, the beam spreading is found to be pronounced for larger variance of displacement parameter that can be related to the fact that the off-axis Gaussian beam wave having a Gaussian distributed random displacement parameter has similar characteristics as that of a spatially partially coherent laser beam. Also, the beam spreads more in turbulence than in free space when the other source and medium parameters are kept the same.

The average receiver intensity profile becomes smaller along the slanted receiver axis as the variance of the displacement parameter increases, the reduction being larger at the origin of the receiver plane.

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